

THE EDGE ECCENTRIC CONNECTIVITY INDEX OF SOME CHAIN SILICATES NETWORKS

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Abstract. A topological index is a numerical descriptor of the molecular structure derived from the corresponding molecular graph. Various topological indices are widely used for quantitative structure property relationship (QSPR) and quantitative structure activity relationship (QSAR) studies. In this paper, exact formulas for the edge eccentric connectivity index of single and double chain silicates networks have been derived.

Keywords: Distance, eccentricity, edge eccentric connectivity index, chain silicates networks.

AMS Subject Classification: 05C12, 05C85, 68R10, 92E10.

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1. Introduction

Graph theory has seen an explosive growth due to interaction with areas such as information science, mathematics, chemistry, etc. Especially, it has become one of the most powerful mathematical tools in the analysis and study of the chemical sciences [18]. A chemical graph is a graph such that each vertex represents an atom of the molecule, and represents covalent bonds between atoms by edges of the corresponding vertices [1]. Furthermore, the graph theory has successfully provided chemists with a variety of very useful tools, namely, the topological index. A topological index is a numerical value associated with chemical constitution purporting for the correlation of a chemical structure with various physical properties, chemical reactivity or biological activity. Research on the topological indices has been intensively rising recently. There are numerous topological descriptors that have found some applications in theoretical chemistry, especially in QSPR/QSAR research [2-4, 6, 7, 9, 12].

Let $G = (V, E)$ be a simple undirected graph of order n and size m . We begin by recalling some standard definitions that we need throughout this paper. For any vertex $v \in V$, the open neighborhood of v is $N_G(v) = \{u \in V \mid uv \in E\}$ and closed neighborhood of v is $N_G[v] = N_G(v) \cup \{v\}$. The degree of vertex v in G denoted by $deg_G(v)$, that is the size of its open neighborhood. The distance $d_G(u, v)$ between two vertices u and v in G is the length of a shortest path between them [5,21]. The eccentricity value of the vertex $u \in V$ denoted by $\varepsilon_G(u)$, that is the

largest between vertex u and any other vertex v of G , $\varepsilon_G(u) = \max_{v \in V(G)} d(u, v)$. Let $f = uv \in E$. Then, the degree of the edge f , denoted by $\deg_G(f)$, is defined to be $\deg_G(f) = \deg_G(u) + \deg_G(v) - 2$. Let $f_1 = u_1v_1$ and $f_2 = u_2v_2$ be two edges in E . The distance between f_1 and f_2 , denoted by $d_G(f_1, f_2)$, is defined to $d_G(f_1, f_2) = \min\{d_G(u_1, u_2), d_G(u_1, v_2), d_G(v_1, u_2), d_G(v_1, v_2)\}$. The eccentricity value of the edge $f \in E$, denoted by $\varepsilon_G(f)$, is defined as $\varepsilon_G(f) = \max\{d_G(f, e) \mid e \in E\}$ [5, 21].

The first topological index namely wiener index in chemistry is developed by the chemist Harold Wiener [22]. The wiener index aims to sum of the half of distances between every pair of vertices of G and it is defined as:

$$W(G) = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n d_G(v_i, v_j).$$

There are a lot of topological indices were introduced after defining the wiener index. More recently, a new topological index called eccentric connectivity index has been investigated. The eccentric connectivity index $\xi^c(G)$ was defined by Sharma et al [15] and has been further studied by some authors [10, 11, 15, 23]. The eccentric connectivity index $\xi^c(G)$ for any graph G is defined as:

$$\xi^c(G) = \sum_{u \in V(G)} \varepsilon_G(u) \deg_G(u).$$

After, a new topological index, *edge eccentric connectivity index*, has been studied. This index was introduced by Xu et al. [20] and has been further studied by Odabas [4, 13], Turaci et al. [19] and Aslan [2]. The edge eccentric connectivity index of a graph G denoted by $\xi_e^c(G)$, is defined as:

$$\xi_e^c(G) = \sum_{f \in E(G)} \varepsilon_G(f) \deg_G(f),$$

where $\varepsilon_G(f)$ is the eccentricity value and $\deg_G(f)$ is the degree of an edge f in the graph G . The eccentric connectivity index and the edge eccentric connectivity index are the distance-related topological invariants whose potential of predicting biological activity of the certain classes of chemical compounds made them very attractive for use in QSAR/QSPR studies [13, 18, 19].

The silicates are the largest, the most interesting and the most complicated classes of minerals so far [8, 14]. Furthermore, the silicates are obtained by fusing metal oxides or metal carbonates with sand, also they are building blocks of the common rock-forming minerals [14, 16]. The tetrahedron SiO_4 is a basic unit of silicates. Almost all silicates contain SiO_4 tetrahedral [14, 17]. In the chemistry, the corner vertices of SiO_4 tetrahedran represent oxygen ions and the center vertex represents the silicon ion. In the graph theory, we call the corner vertices as oxygen nodes and the center vertex as silicon node [14]. We display a SiO_4 tetrahedron in Figure 1.

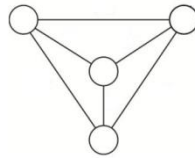


Figure 1. A SiO_4 tetrahedron in which corner vertices are oxygen vertices and central vertex is silicon vertex.

Some of the structural units found in silicates which are called orthosilicates, pyrosilicates and chain silicates are shown in Figure 2.



Figure 2. Different kinds of silicates.

2. The edge eccentric connectivity index of some chain silicates networks

Definition 2.1. [8] A chain silicates network of length n symbolizes as CS_n is obtained by arranging n tetrahedra linearly. The number of vertices in CS_n with $n \geq 1$ is $3n+1$ and number of edges is $6n$. A chain silicates network of length five is shown in Figure 3.

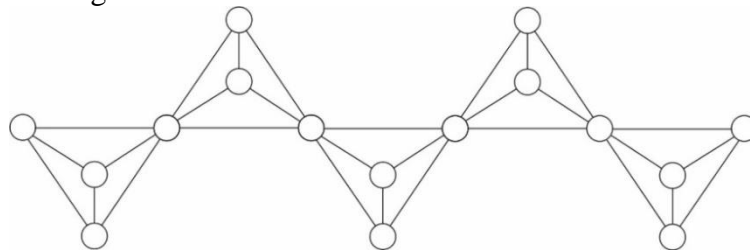


Figure 3. The chain silicates network CS_5 .

Definition 2.2. We define double chain silicates network of length n as follows: A double chain silicates network of length n symbolizes as DCS_n , also it consists of two condensed identical silicates chains. The number of vertices in DCS_n with $n \geq 1$ is $\frac{11n + 7/2 + 1/2(-1)^n}{2}$ and the number of edges is $12n$. A double chain silicates network of length five is shown in Figure 4.

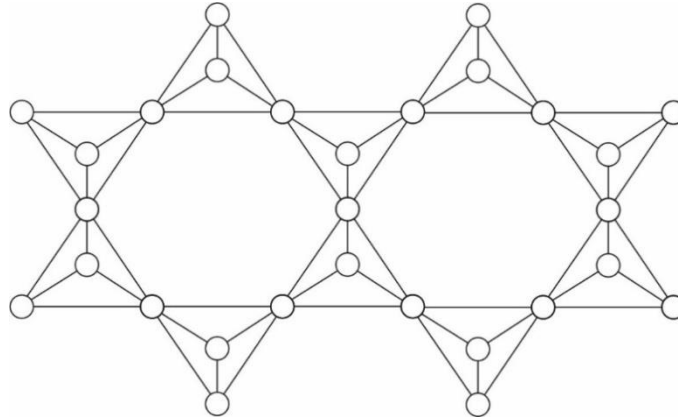


Figure 4. The double chain silicates network DCS_5 .

Theorem 2.1. Let CS_n be a chain silicates network of length $n \geq 2$. Then,

$$\xi_e^c(CS_n) = \begin{cases} \frac{1}{2}(63n^2 - 42n + 19) & , \text{if } n \text{ is odd;} \\ \frac{1}{2}(63n^2 - 42n + 12) & , \text{if } n \text{ is even.} \end{cases}$$

Proof. We first label the edges of the chain silicates network CS_n as in Figure 5.

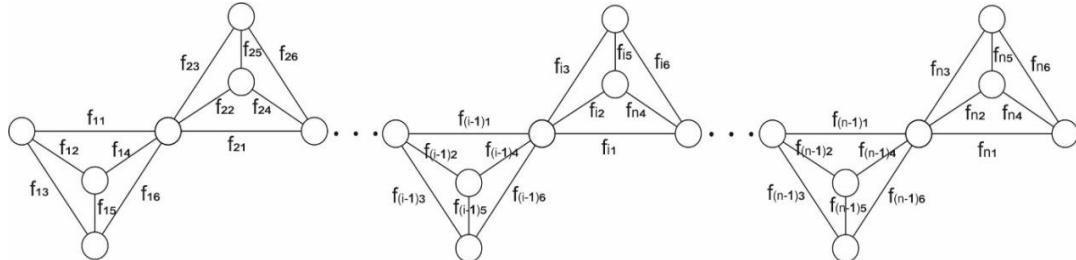


Figure 5. Labeling of edges in a chain silicates network CS_n .

The degrees of every edge in the chain silicates network CS_n of length n are as follows:

- $\deg_{CS_n}(f_{i1}) = 10$, where $2 \leq i \leq n-1$,
- $\deg_{CS_n}(f_{i2}) = \deg_{CS_n}(f_{i3}) = 7$, where $2 \leq i \leq n$,
- $\deg_{CS_n}(f_{i4}) = \deg_{CS_n}(f_{i6}) = 7$, where $1 \leq i \leq n-1$,
- $\deg_{CS_n}(f_{i5}) = 4$, where $1 \leq i \leq n$,
- $\deg_{CS_n}(f_{n1}) = \deg_{CS_n}(f_{n1}) = 7$,
- $\deg_{CS_n}(f_{12}) = \deg_{CS_n}(f_{13}) = \deg_{CS_n}(f_{n4}) = \deg_{CS_n}(f_{n6}) = 4$.

The eccentricity values of the edges in chain silicates network CS_n of length n can be derived as follows:

$$\left. \begin{aligned} \varepsilon_{CS_n}(f_{i2}) = \varepsilon_{CS_n}(f_{i3}) = \varepsilon_{CS_n}(f_{i5}) = n - i + 1 \\ \varepsilon_{CS_n}(f_{i1}) = \varepsilon_{CS_n}(f_{i4}) = \varepsilon_{CS_n}(f_{i6}) = n - i \end{aligned} \right\}, \text{ where } i \leq \left\lfloor \frac{n}{2} \right\rfloor.$$

We have two cases depending on n .

Case 1. n is even.

$$\begin{aligned} \xi_e^c(CS_n) &= \sum_{i=1}^n \left(\sum_{j=1}^6 \varepsilon_{CS_n}(f_{ij}) \deg_{CS_n}(f_{ij}) \right) \\ &= 2 \sum_{i=1}^{n/2} \left(\sum_{j=1}^6 \varepsilon_{CS_n}(f_{ij}) \deg_{CS_n}(f_{ij}) \right) \\ &= 2 \left(\sum_{i=1}^{n/2} \left(((n-i)10^*) + (2((n-i+1)7^*)) + (2((n-i)7^*)) + ((n-i+1)4) \right) \right) \\ &\quad - ((3.4.n) + (3.2.(n-1))) \\ &= 2 \left(\sum_{i=1}^{n/2} [42n - 42i + 18] \right) - (18n - 6) \\ &= \frac{1}{2} (63n^2 - 42n + 12). \end{aligned}$$

Case 2. n is odd.

$$\xi_e^c(CS_n) = \sum_{i=1}^n \left(\sum_{j=1}^6 \varepsilon_{CS_n}(f_{ij}) \deg_{CS_n}(f_{ij}) \right)$$

Let $k = (n+1)/2$.

$$= 2 \sum_{i=1}^{(n-1)/2} \left(\sum_{j=1}^6 \varepsilon_{CS_n}(f_{ij}) \deg_{CS_n}(f_{ij}) \right) + \sum_{j=1}^6 \varepsilon_{CS_n}(f_{kj}) \deg_{CS_n}(f_{kj})$$

Clearly, we get

$$\begin{aligned} \varepsilon_{CS_n}(f_{k2}) = \varepsilon_{CS_n}(f_{k3}) = \varepsilon_{CS_n}(f_{k4}) = \varepsilon_{CS_n}(f_{k5}) = \varepsilon_{CS_n}(f_{k6}) = (n+1)/2, \\ \varepsilon_{CS_n}(f_{k1}) = (n-1)/2. \end{aligned}$$

Thus,

$$\begin{aligned} &= 2 \left(\sum_{i=1}^{(n-1)/2} \left(((n-i)10^*) + (2((n-i+1)7^*)) + (2(n-i)7^*) + ((n-i+1)4) \right) \right) \\ &\quad - ((3.4.n) + (3.2.(n-1))) + \left(4.7. \left(\frac{n+1}{2} \right) + 4. \left(\frac{n+1}{2} \right) + 10. \left(\frac{n-1}{2} \right) \right) \\ &= 2 \left(\sum_{i=1}^{(n-1)/2} [42n - 42i + 18] \right) - (18n - 6) + (21n + 11) \\ &= \frac{1}{2} (63n^2 - 42n + 19). \end{aligned}$$

The numbers with star in $\xi_e^c(CS_n)$ are the degrees of $f_{i1}, f_{i2}, f_{i3}, f_{i4}$ and f_{i6} for $i \leq \lfloor \frac{n}{2} \rfloor$. But, $\deg_{CS_n}(f_{11}) = \deg_{CS_n}(f_{n1}) = 7$ and $\deg_{CS_n}(f_{12}) = \deg_{CS_n}(f_{13}) = \deg_{CS_n}(f_{n4}) = \deg_{CS_n}(f_{n6}) = 4$ are obtained. Therefore, we subtract $(18n-6)$ from the $\xi_e^c(CS_n)$ in the Cases 1 and 2.

The proof is completed.

Theorem 2.2. Let DCS_n be a double chain silicates network of length $n \geq 4$. Then,

$$\xi_e^c(DCS_n) = \begin{cases} \left[-\frac{37}{2} + 40n + \frac{93n^2}{2} + 42n \left\lfloor \frac{n-1}{4} \right\rfloor - 84 \left\lfloor \frac{n-1}{4} \right\rfloor^2 + (102 + 88n) \left\lfloor \frac{n}{4} \right\rfloor \right. \\ \left. - 176 \left\lfloor \frac{n}{4} \right\rfloor^2 - 84 \left\lfloor \frac{n}{4} \right\rfloor + 56n \left\lfloor \frac{n}{4} \right\rfloor - 112 \left\lfloor \frac{n}{4} \right\rfloor^2 \right], & \text{if } n \text{ is odd;} \\ \left[-18 + 31n + \frac{93n^2}{2} + 102(n+1) \left\lfloor \frac{n}{4} \right\rfloor - 204 \left\lfloor \frac{n}{4} \right\rfloor^2 \right. \\ \left. + 84(n-1) \left\lfloor \frac{n}{4} \right\rfloor - 168 \left\lfloor \frac{n}{4} \right\rfloor^2 \right], & \text{if } n \text{ is even.} \end{cases}$$

Proof. We will consider two cases in which n is even and odd, separately.

Case 1. n is even.

We first label the edges of the double chain silicates network DCS_n as in Figure 6. Note that we need to compute the degree and the eccentricity value of each edge in the first and second layer of the silicates, that is, the edges f and f' , since the other layers are symmetric. According to our labeling, if n is even, then the number of silicates in the first and the second layer is the same which is $n/2$, however, the number of silicates in the first and the second layer is $\lfloor n/2 \rfloor$ and $\lceil n/2 \rceil$, respectively, when n is odd.

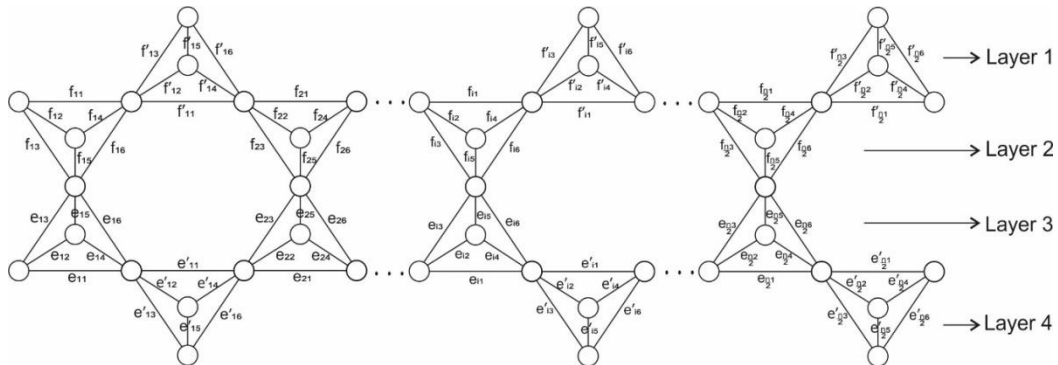


Figure 6. Labeling of the edges in a double chain silicates network DCS_n , where n is even.

The degrees of the edges in a double chain silicates network DCS_n of even length n are as follows for the edges in the first layer;

$$\left. \begin{aligned} \deg_{DCS_n}(f'_{i1}) &= 10 \\ \deg_{DCS_n}(f'_{i4}) &= \deg_{DCS_n}(f'_{i6}) = 7 \end{aligned} \right\}, \text{where } 1 \leq i < \frac{n}{2}$$

$$\left. \begin{aligned} \deg_{DCS_n}(f'_{i2}) &= \deg_{DCS_n}(f'_{i3}) = 7 \\ \deg_{DCS_n}(f'_{i5}) &= 4 \end{aligned} \right\}, \text{where } 1 \leq i \leq \frac{n}{2}$$

$$\deg_{DCS_n}(f'_{(n/2)1}) = 7, \deg_{DCS_n}(f'_{(n/2)4}) = \deg_{DCS_n}(f'_{(n/2)6}) = 4.$$

For the edges in the second layer;

$$\left. \begin{aligned} \deg_{DCS_n}(f_{i1}) &= \deg_{DCS_n}(f_{i3}) = 10 \\ \deg_{DCS_n}(f_{i2}) &= 7 \end{aligned} \right\}, \text{where } 1 < i \leq \frac{n}{2}$$

$$\left. \begin{aligned} \deg_{DCS_n}(f_{i4}) &= \deg_{DCS_n}(f_{i5}) = 7 \\ \deg_{DCS_n}(f_{i6}) &= 10 \end{aligned} \right\}, \text{where } 1 \leq i \leq \frac{n}{2}$$

$$\deg_{DCS_n}(f_{12}) = \deg_{DCS_n}(f_{13}) = 7, \deg_{DCS_n}(f_{12}) = 4.$$

The eccentricity values of the edges in a double chain silicates network DCS_n of even length n can be derived as follows for the edges in the first layer;

$$\left. \begin{aligned} \varepsilon_{DCS_n}(f'_{i2}) &= \varepsilon_{DCS_n}(f'_{i3}) = \varepsilon_{DCS_n}(f'_{i5}) = n + 2 - 2i \\ \varepsilon_{DCS_n}(f'_{i1}) &= \varepsilon_{DCS_n}(f'_{i4}) = \varepsilon_{DCS_n}(f'_{i6}) = n + 1 - 2i \end{aligned} \right\}, \text{where } i \leq \left\lfloor \frac{n}{4} \right\rfloor$$

$$\left. \begin{aligned} \varepsilon_{DCS_n}(f'_{i1}) &= \varepsilon_{DCS_n}(f'_{i2}) = \varepsilon_{DCS_n}(f'_{i3}) = 2i \\ \varepsilon_{DCS_n}(f'_{i4}) &= \varepsilon_{DCS_n}(f'_{i5}) = \varepsilon_{DCS_n}(f'_{i6}) = 2i + 1 \end{aligned} \right\}, \text{where } \left\lfloor \frac{n}{4} \right\rfloor + 1 \leq i \leq \frac{n}{2}.$$

For the edges in the second layer;

$$\left. \begin{aligned} \varepsilon_{DCS_n}(f_{i1}) &= \varepsilon_{DCS_n}(f_{i3}) = \varepsilon_{DCS_n}(f_{i4}) = \varepsilon_{DCS_n}(f_{i5}) = \varepsilon_{DCS_n}(f_{i6}) = n + 2 - 2i \\ \varepsilon_{DCS_n}(f_{i2}) &= n + 3 - 2i \end{aligned} \right\},$$

$$\text{where } i \leq \left\lfloor \frac{n}{4} \right\rfloor$$

$$\left. \begin{aligned} \varepsilon_{DCS_n}(f_{i1}) &= \varepsilon_{DCS_n}(f_{i2}) = \varepsilon_{DCS_n}(f_{i3}) = \varepsilon_{DCS_n}(f_{i5}) = \varepsilon_{DCS_n}(f_{i6}) = 2i - 1 \\ \varepsilon_{DCS_n}(f_{i4}) &= 2i \end{aligned} \right\},$$

$$\text{where } \left\lfloor \frac{n}{4} \right\rfloor + 1 \leq i \leq \frac{n}{2}.$$

Since the bottom two layers are symmetric with respect to the top two layers we multiply the eccentric connectivity index of the graph at the top two layers by two to calculate the eccentric connectivity index of a DCS_n . That is,

$$\xi_e^c(DCS_n) = 2x \sum_{i=1}^{n/2} \left(\sum_{j=1}^6 \varepsilon_{DCS_n}(f'_{ij}) \deg_{DCS_n}(f'_{ij}) + \varepsilon_{DCS_n}(f_{ij}) \deg_{DCS_n}(f_{ij}) \right).$$

The first and the second term of the inner summation belong to the first and second layer of the DCS_n , respectively. Now, we investigate the eccentric

connectivity index of the edges in the first layer and denote it by $EC1$ then of the second layer and denote it by $EC2$.

$$\begin{aligned}
 EC1 &= \sum_{i=1}^{n/2} \left(\sum_{j=1}^6 \varepsilon_{DCS_n}(f'_{ij}) \deg_{DCS_n}(f'_{ij}) \right) \\
 &= \sum_{i=1}^{\lfloor n/4 \rfloor} \left(\sum_{j=1}^6 \varepsilon_{DCS_n}(f'_{ij}) \deg_{DCS_n}(f'_{ij}) \right) + \sum_{i=\lfloor n/4 \rfloor + 1}^{n/2} \left(\sum_{j=1}^6 \varepsilon_{DCS_n}(f'_{ij}) \deg_{DCS_n}(f'_{ij}) \right) \\
 &= \sum_{i=1}^{\lfloor n/4 \rfloor} \left((n+1-2i)10 + (n+2-2i)7 + (n+2-2i)7 + (n+1-2i)7 + (n+2-2i)4 \right. \\
 &\quad \left. + (n+1-2i)7 \right) \\
 &\quad + \sum_{i=\lfloor n/4 \rfloor + 1}^{n/2} \left(2i(10^* + 7 + 7) + (2i+1)(7^* + 4 + 7^*) \right) - (9n+6) \\
 &= -6 + 21n + \frac{21n^2}{2} + 42(n-1) \left\lfloor \frac{n}{4} \right\rfloor - 84 \left\lfloor \frac{n}{4} \right\rfloor^2.
 \end{aligned}$$

The numbers with star in EC1 are the degrees of $f'_{i1}, f'_{i4}, f'_{i6}$ for $\lfloor n/4 \rfloor + 1 \leq i \leq n/2$. However, $\deg_{DCS_n}(f'_{(n/2)1}), \deg_{DCS_n}(f'_{(n/2)4})$ and $\deg_{DCS_n}(f'_{(n/2)6})$ are 7, 4 and 4, respectively. Therefore, we subtract $(9n+6)$ from the $EC1$.

$$\begin{aligned}
 EC2 &= \sum_{i=1}^{n/2} \left(\sum_{j=1}^6 \varepsilon_{DCS_n}(f_{ij}) \deg_{DCS_n}(f_{ij}) \right) \\
 &= \sum_{i=1}^{\lfloor n/4 \rfloor} \left(\sum_{j=1}^6 \varepsilon_{DCS_n}(f_{ij}) \deg_{DCS_n}(f_{ij}) \right) + \sum_{i=\lfloor n/4 \rfloor + 1}^{n/2} \left(\sum_{j=1}^6 \varepsilon_{DCS_n}(f_{ij}) \deg_{DCS_n}(f_{ij}) \right) \\
 &= \sum_{i=1}^{\lfloor n/4 \rfloor} \left((n+2-2i)10^* + (n+3-2i)7^* + (n+2-2i)10^* + (n+2-2i)(7+7+10) \right) - (3n + (3n+3) + 3n) \\
 &\quad + \sum_{i=\lfloor n/4 \rfloor + 1}^{n/2} \left((2i-1)10 + (2i-1)7 + (2i-1)10 + 2i \cdot 7 + (2i-1)(7+10) \right) \\
 &= -3 - \frac{11n}{2} + \frac{51n^2}{4} + 51(n+1) \left\lfloor \frac{n}{4} \right\rfloor - 102 \left\lfloor \frac{n}{4} \right\rfloor^2.
 \end{aligned}$$

Finally, if we sum EC1 and EC2 and multiply the summation by two, then we obtain the following formula which proves the Case 1:

$$\xi_e^c(DCS_n) = -18 + 31n + \frac{93n^2}{2} + 102(n+1) \left\lfloor \frac{n}{4} \right\rfloor - 204 \left\lfloor \frac{n}{4} \right\rfloor^2 + 84(n-1) \left\lfloor \frac{n}{4} \right\rfloor - 168 \left\lfloor \frac{n}{4} \right\rfloor^2.$$

Case 2. n is odd.

Similar to that used to prove Case 1.

The proof is completed.

3. Conclusion

Research on the topological indices of graphs has been intensively rising recently. The most common usage areas of these indices are networks and

measurements of the durability of chemical graphs. In this paper, exact formulas for the edge eccentric connectivity index of two different types of silicates networks have been derived. Besides, we tested and verified the formulas of the theorems for several CS_n and DCS_n using Mathematica. We gave a Mathematica code in Appendix that computes the eccentric connectivity index for not only silicates networks but any undirected graph.

In future, we are interested to study different topological indices of silicate networks and compare the results with the edge eccentric connectivity index.

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Appendix

The code below runs in Mathematica 10.2 for a given undirected graph G which is represented by adjacency list.

```
(* The double chain silicates network DSCn, where n=8*)
G = Graph[{1 ↔ 2, 1 ↔ 3, 1 ↔ 4, 2 ↔ 3, 2 ↔ 4, 3 ↔ 4, 4 ↔ 5, 4 ↔ 6, 4 ↔ 7, 5 ↔ 6, 5 ↔ 7, 6 ↔ 7,
7 ↔ 8, 7 ↔ 9, 7 ↔ 10, 8 ↔ 9, 8 ↔ 10, 9 ↔ 10, 10 ↔ 11, 10 ↔ 12, 10 ↔ 13, 11 ↔ 13, 11 ↔ 12,
12 ↔ 13, 14 ↔ 11, 14 ↔ 15, 14 ↔ 16, 11 ↔ 15, 11 ↔ 16, 15 ↔ 16, 16 ↔ 17, 16 ↔ 18, 16 ↔ 19,
17 ↔ 18, 17 ↔ 19, 18 ↔ 19, 19 ↔ 5, 19 ↔ 20, 19 ↔ 21, 20 ↔ 21, 20 ↔ 5, 21 ↔ 5, 21 ↔ 22,
21 ↔ 23, 21 ↔ 24, 22 ↔ 23, 22 ↔ 24, 23 ↔ 24, 12 ↔ a2, 12 ↔ a3, 12 ↔ a4, a2 ↔ a3, a2 ↔ a4,
a3 ↔ a4, a4 ↔ a5, a4 ↔ a6, a4 ↔ a7, a5 ↔ a6, a5 ↔ a7, a6 ↔ a7, a7 ↔ a8, a7 ↔ a9, a7 ↔ a10,
a8 ↔ a9, a8 ↔ a10, a9 ↔ a10, a10 ↔ a11, a10 ↔ a12, a10 ↔ a13, a11 ↔ a13, a11 ↔ a12,
a12 ↔ a13, a14 ↔ a11, a14 ↔ a15, a14 ↔ a16, a11 ↔ a15, a11 ↔ a16, a15 ↔ a16, a16 ↔ a17,
a16 ↔ a18, a16 ↔ a19, a17 ↔ a18, a17 ↔ a19, a18 ↔ a19, a19 ↔ a5, a19 ↔ a20, a19 ↔ a21,
a20 ↔ a21, a20 ↔ a5, a21 ↔ a5, a21 ↔ a22, a21 ↔ a23, a21 ↔ 15, a22 ↔ a23, a22 ↔ 15, a23 ↔ 15}];
A = Normal[AdjacencyMatrix[G]];
n = Length[A];

ecconindex = 0;
For[i = 1, i ≤ n, i++,
  For[j = i + 1, j ≤ n, j++,
    If[A[[i]][[j]] == 1,
      max = 0;
      deg = Total[A[[i]] + A[[j]]] - 2;
      For[k = 1, k ≤ n, k++,
        For[l = k + 1, l ≤ n, l++,
          If[A[[k]][[l]] == 1,
            min = Min[Length[GraphPath[A, i, k]] - 1, Length[GraphPath[A, i, l]] - 1,
              Length[GraphPath[A, j, k]] - 1, Length[GraphPath[A, j, l]] - 1];
            If[max < min, max = min]
          ]
        ]
      ]
    ]
  ]
  (*Print["(", i, ", ", j, ")", "\t", "ec=", max, "\t degree=", deg];*)
  ecconindex = ecconindex + deg * max;
]
]
]
Print["The edge eccentric connectivity index=", ecconindex]
The edge eccentric connectivity index=4730
```